Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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To cite this Article Ledney, Mykhaylo and Pinkevych, Igor(2007) 'Influence of anchoring at a nematic cell surface on threshold spatially periodic reorientation of a director', Liquid Crystals, 34: 5, 577 — 583
To link to this Article: DOI: 10.1080/02678290500248160
URL: http://dx.doi.org/10.1080/02678290500248160

Liquid Crystals, Vol. 34, No. 5, May 2007, 577–583

Influence of anchoring at a nematic cell surface on threshold spatially periodic reorientation of a director

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(*Received 6 January 2005; accepted in revised form 29 April 2005*)

We have analysed the influence of surface director anchoring in a planar flexoelectric nematic cell on the threshold spatially periodic reorientation of the director in an external dc electric field. By minimizing the free energy of the nematic cell we obtained the equations for a director and numerically solved them in the one elastic constant approximation. The dependences of the threshold electric field and the spatial period of director structure on the azimuthal and polar anchoring energy, as well as the flexoelectric parameters, are determined. It is shown that the domain of the flexoelectric parameter values, at which the spatially periodic reorientation of a director takes place, increases with decreasing azimuthal anchoring energy and increasing polar anchoring energy.

1. Introduction

Director reorientation phenomena in nematic liquid crystals (LCs) in electric or magnetic fields, in particular the director threshold reorientation, are often used in different electro-optical devices [1]. Mostly, the uniform (in a cell plane) reorientation of a director has attracted attention. However, in some cases the spatially periodic structure of a director field can arise in a cell plane at the director threshold reorientation. This phenomenon was considered by Bobylev and Pikin [2] and Bobylev *et al.* [3] in the case of a flexoelectric nematic LC and at the initial planar orientation of a director in a cell. Lonberg and Meyer [4] have shown that spatially periodic threshold reorientation of a director can also arise in a non-flexoelectric LC if the Frank elastic constants of the LC satisfy the inequality $K_2<rK_1$, where $r\sim0.3$. Galatola *et al.* [5] have studied the general behaviour of planar nematic layer in the external electric field taking into account the flexoelectricity and dielectric anisotropy as well. In these papers, the model of an infinitely rigid director anchored at the nematic cell surface was used.

Despite orientational transitions in nematic cells being bulk effects, their characteristics as the threshold field value and the director reorientation degree can essentially depend on the strength of director anchoring at the cell surface. In particular, Oldano [6] and Miraldi *et al.* [7] noticed that the type of director field structure arising at the orientational transition depends on the strength of director anchoring at the cell surface. Romanov and Sklyarenko [8] have studied the influence of the strength of director anchoring at the cell surface on the threshold and spatial period of director reorientation in the homeotropic flexoelectric nematic cell.

In present paper we study theoretically the spatially periodic threshold reorientation of a director under an external dc electric field in a planar flexoelectric nematic cell with arbitrary strength of director anchoring at the cell surface. In this case the flexoelectric polarization appears not only in the differential equations for a director, since it takes place at the the threshold orientational transition in the homeotropic cell, but also in the boundary conditions for the director as well. Besides, director anchoring at the cell surface is now described by two parameters, the azimuthal and polar anchoring energies, in accordance with the azimuthal and polar director deviations in the planar nematic cell.

The paper is arranged as follows. In section 2 we present the basic equations that manage the director in the planar flexoelectric nematic cell in an external electric field and give the general solution to the equations in the cell bulk. In section 3 we take into account the boundary conditions and numerically (analytically in the limiting cases) study the dependence of the threshold field value and the spatial period of director structure on both the director azimuthal anchoring energy and the flexoelectric parameter value. The influence of the director polar anchoring energy on the threshold field and the director period is analysed in section 4. In section 5 we give the brief conclusions that follow from the results obtained in the paper.

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Liquid Crystals
ISSN 0267-8292 print/ISSN 1366-5855 online © 2007 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/02678290500248160

2. Basic equations

Let a plane parallel flexoelectric nematic cell be bounded by the planes $z=-L/2$ and $z=L/2$ with initial planar director oriented along the axis Ox. The cell is placed into the spatially uniform dc electric field E directed along the axis Oz.

The free energy of a nematic cell in the one elastic constant approximation can be written as follows [9]

$$\begin{gathered} F=F_{el}+F_E+F_d+F_S, \\ F_{el}=\frac{K}{2}\int\limits_V\left\{(\mathrm{divn})^2+(\mathrm{rotn})^2\right\}\mathrm{d}V, \\ F_E=-\frac{\varepsilon_a}{8\pi}\int\limits_V(\mathrm{nE})^2\mathrm{d}V, \\ F_d=-\int\limits_V\{e_1(\mathrm{nE})\mathrm{divn}+e_3([\mathrm{rotn}\times\mathrm{n}]\mathrm{E})\}\mathrm{d}V, \\ F_S=-\frac{W_\varphi}{2}\int\limits_{S_{1,2}}\cos^2\varphi\,\mathrm{d}S-\frac{W_\theta}{2}\int\limits_{S_{1,2}}\cos^2\theta\,\mathrm{d}S, \\ W_\varphi>0,\ W_\theta>0, \end{gathered} \tag{1}$$

!

where F_{el}, F_E, F_d are the contributions to the free energy from the Frank elastic energy, the anisotropic and flexoelectric interaction of the LC with electric field, respectively, F_S is the surface free energy of the LC written in the model of Rapini–Papoular [10], $\boldsymbol{n}$ is a director, $\varepsilon_a=\varepsilon_{\|}-\varepsilon_{\perp}>0$ is the static dielectric anisotropy of the LC, e_1, e_3 are the flexoelectric coefficients, and W_θ, W_φ, θ, φ denote, respectively, the energies of polar and azimuthal anchoring of a director at the cell surface and the director deviation angles in the planes xOz and xOy.

Strictly speaking, one must also take into account the fact that the flexoelectric polarization induces an electric field, which couples with the flexoelectric polarization itself, and include the corresponding term into equation (1) for the free energy. As was shown by Bobylev and Pikin [2], this term renormalizes the Frank elastic constant K_i only on the value of order $0.01K_i$. On the other hand, as was noted by Madhusudana and Durand [11] and Barbero *et al.* [12], at the director deformations, which are larger than a Debye screening length (the last is of order one micron in the commercial LCs [13]) the flexoelectric contribution to the free energy can be described by a linear coupling of the flexoelectric polarization with an external field, just that very case we will investigate in this paper.

To consider the reorientation of a director, which is spatially periodic along the axis Oy one can seek a director in the form

$$\begin{aligned} \mathbf{n}=\mathbf{i}&\cos\theta(y,z)\cos\varphi(y,z) \\ &+\mathbf{j}\cos\theta(y,z)\sin\varphi(y,z)+\mathbf{k}\sin\theta(y,z), \end{aligned} \tag{2}$$

where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the Cartesian unit vectors.

Substituting the equation (2) into equation (1) and minimizing the obtained expression for the free energy we get in the linear θ and φ approximation the next stationery equations

$$\begin{aligned} &K\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)+\in E^2\theta+eE\frac{\partial\varphi}{\partial y}=0, \\ &K\left(\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right)-eE\frac{\partial\theta}{\partial y}=0, \end{aligned} \tag{3}$$

and the boundary conditions

$$\begin{aligned} &\left[(W_\theta\mp e_oE)\theta\pm K\left(\frac{\partial\theta}{\partial z}+\frac{\partial\varphi}{\partial y}\right)\right]_{z=\pm L/2}=0, \\ &\left[W_\varphi\cdot\varphi\pm K\left(\frac{\partial\varphi}{\partial z}-\frac{\partial\theta}{\partial y}\right)\right]_{z=\pm L/2}=0, \end{aligned} \tag{4}$$

where we have introduced the designations $\in=\frac{\varepsilon_a}{4\pi}$, $e=e_1-e_3\neq 0$, $e_o=e_1+e_3$.

Taking into account the symmetry of equations (3) one can seek the solution to these equations in the form

$$\theta(y,z)=\theta_0\cos(qy)e^{ipz},\qquad \varphi(y,z)=\varphi_0\sin(qy)e^{ipz}. \tag{5}$$

Substituting the equation (5) into equation (3) we obtain the system of two homogeneous algebraic equations to determine the unknown coefficients θ_0 and φ_0. The condition of non-trivial solution to the system gives us the equation to determine the unknown quantity p

$$\left(Kq^2+Kp^2\right)^2-\in E^2\left(Kq^2+Kp^2\right)-(eEq)^2=0. \tag{6}$$

From this system of equations we also obtain the ratio

$$\frac{\varphi_0}{\theta_0}=\frac{eEq}{Kq^2+Kp^2}\equiv\alpha(p). \tag{7}$$

Solving the equation (6) we get $p=\pm p_1,\ \pm p_2$, where

$$\begin{aligned} p_1&=\sqrt{\frac{1}{2K}\left(\in E^2+\sqrt{(\in E^2)^2+(2eEq)^2}\right)-q^2}, \\ p_2&=\sqrt{\frac{1}{2K}\left(\in E^2-\sqrt{(\in E^2)^2+(2eEq)^2}\right)-q^2}. \end{aligned} \tag{8}$$

In these designations the general solution to equations (3) takes the form

$$\begin{aligned}\theta(y,z)&=\cos(qy)(a_1\cos p_1z+a_2\sin p_1z\\&\quad+b_1\cos p_2z+b_2\sin p_2z),\\\varphi(y,z)&=sin(qy)[\alpha_1(a_1\cos p_1z+a_2\sin p_1z)\\&\quad+\alpha_2(b_1\cos p_2z+b_2\sin p_2z)],\end{aligned}\tag{9}$$

where

$$\begin{aligned}\alpha_1&=\alpha(p=\pm p_1)=\frac{2eEq}{\in E^2+\sqrt{(\in E^2)^2+(2eEq)^2}},\\\alpha_2&=\alpha(p=\pm p_2)=\frac{2eEq}{\in E^2-\sqrt{(\in E^2)^2+(2eEq)^2}}\end{aligned}\tag{10}$$

and a_i, b_i (i=1, 2) are the constants which have to be determined from the boundary conditions of equation (4). It should also be noted that, as it follows from the definition in equation (8), the parameter p_2 is always an imaginary value while p_1 can take, in principle, both the real and imaginary values.

3. Influence of the azimuthal anchoring energy

Let us suppose the director polar anchoring energy to be infinite, $W_\theta=\infty$, whereas the director azimuthal anchoring energy, W_φ, can take the arbitrary values. In this case the boundary conditions of equation (4) take the form

$$\begin{aligned}&\theta|_{z=\pm L/2}=0,\\&\left[W_\varphi\cdot\varphi\pm K\frac{\partial\varphi}{\partial z}\right]_{z=\pm L/2}=0.\end{aligned}\tag{11}$$

Substituting equation (9) into the boundary conditions of equation (11) one can obtain the system of four homogeneous algebraic equations to find the coefficients a_i, b_i (i=1, 2). The condition of non-trivial solution to the system takes the form

$$\begin{aligned}&\cos\frac{p_1L}{2}\cos\frac{p_2L}{2}\left\{2W_\varphi\sqrt{(\in E^2)^2+(2eEq)^2}+\right.\\&+Kp_1\left(\in E^2-\sqrt{(\in E^2)^2+(2eEq)^2}\right)\tan\frac{p_1L}{2}-\\&\left.-Kp_2\left(\in E^2+\sqrt{(\in E^2)^2+(2eEq)^2}\right)\tan\frac{p_2L}{2}\right\}=0.\end{aligned}\tag{12}$$

Substituting into equation (12) the expressions given by equation (8) for p_1 and p_2 we can obtain the electric field E as a function of parameter q. To obtain it in a general case of the arbitrary value of W_φ one has to use numerical methods. But in the limiting case of the infinitely rigid director anchoring with the cell surface ($W_\varphi=\infty$) it is more easy first to solve equation (12) that gives us the real value $p_1=\frac{\pi}{L}(2s+1)$, where s is an integer. Then from the first equation (8) we obtain the function $E(q)$, which has the form

$$E^\infty(q)=\frac{q^2+\left(\frac{\pi}{L}(2s+1)\right)^2}{\sqrt{q^2+v\left[q^2+\left(\frac{\pi}{L}(2s+1)\right)^2\right]}}\cdot\frac{K}{e},\tag{13}$$

where we have introduced the flexoelectric parameter

$$v=\frac{\in K}{e^2}>0.$$

The orientation instability threshold is determined by a minimum value of the function $E^\infty(q)$ and equals

$$E_c^\infty=E^\infty\left(q_c^\infty\right)=\frac{2\pi K}{|e|(1+v)L},\tag{14}$$

where the wave number of the arising periodic structure of a director equals

$$q_c^\infty=\frac{\pi}{L}\sqrt{\frac{1-v}{1+v}},\tag{15}$$

in accordance with the results obtained by Bobylev and Pikin [2].

One can see that in the case of an infinitely rigid director anchored at the cell surface the spatially periodic orientational instability of a director only arises at the condition $v<1$.

We can now consider the case of the strong azimuthal anchoring of a director ($\varepsilon_\varphi=\frac{W_\varphi L}{K}\gg1$), seeking a solution to equation (12) in the form $p_1=\frac{\pi}{L}(2s+1)(1-x)$, where $0<x<<1$. Then, with accuracy to the terms linear in $\frac{1}{\varepsilon_\varphi}$, we obtain from equation (12) the expression

$$p_1=\frac{\pi}{L}(2s+1)\left(1+\frac{1}{\varepsilon_\varphi}\frac{\in E^2-\sqrt{(\in E^2)^2+(2eEq)^2}}{\sqrt{(\in E^2)^2+(2eEq)^2}}\right),\tag{16}$$

and then further from the first equation (8) the expression

$$\begin{aligned}&E(q)=E^\infty(q)\\&\left\{1-\frac{2}{\varepsilon_\varphi}\frac{q^2\left(\frac{\pi}{L}(2s+1)\right)^2}{\left(q^2+\left(\frac{\pi}{L}(2s+1)\right)^2\right)\left[q^2+v\left(q^2+\left(\frac{\pi}{L}(2s+1)\right)^2\right)\right]}\right\}.\end{aligned}\tag{17}$$

Minimizing equation (17) gives us the electric field threshold value

$$E_c=E(q_c)=E_c^\infty\left(1-\frac{1-v}{\varepsilon_\varphi}\right),\tag{18}$$

and the corresponding wave number, which equals

$$q_c = q_c^{\infty}\left(1 + \frac{1}{\varepsilon_\varphi}\frac{3v-1}{1-v}\right), \tag{19}$$

if $v<1$ and $\varepsilon_\varphi(1-v)>>1$, or

$$q_c = \frac{\pi}{\sqrt{2}L}\sqrt{\frac{4}{\varepsilon_\varphi} + 1 - v}, \tag{20}$$

if $|1-v|<<1$.

It can be seen from equations (18)–(19) that the threshold field value always increases with increasing azimuthal anchoring energy parameter ε_φ, whereas the corresponding value of a wave number at the flexoelectric parameter values $v<1$, $\varepsilon_\varphi(1-v)>>1$ increases with increasing ε_φ, if $0<v<\frac{1}{3}$, and decreases, if $\frac{1}{3}<v<1$. Besides, as can be seen from equation (20), the periodic structure of a director appears even at the values of the flexoelectric parameter v more than unity, depending on the anchoring energy values, i.e. at $v<1+\frac{4}{\varepsilon_\varphi}$.

Meanwhile, one can also note that the inequality

$$\left.\frac{dE}{dq}\right|_{q=0} < 0 \tag{21}$$

is a general condition for the existence of the threshold spatially periodic structure of a director field. Then differentiating the equation (12) with respect to q in the point $q=0$ and making some algebraic transformations we arrive to the inequality

$$v < 1 + \frac{4}{\varepsilon_\varphi} \equiv v_{th}, \tag{22}$$

which fulfils at the arbitrary values of the anchoring energy parameter ε_φ. Thus the values of the flexoelectric parameter v that determine the domain of existence of the director periodic structure depend essentially on the director azimuthal anchoring energy value. If $\varepsilon_\varphi \to 0$ the value of v can be practically the arbitrary.

Numerically calculated values of the dimensionless threshold electric field, $E'_c = \sqrt{\frac{\epsilon}{K}}E_c L$, and the corresponding wave number, $Q_c = q_c L$, of a director with spatially periodic structure as functions of the parameter ε_φ are shown in figure 1 for different fixed values of the flexoelectric parameter v. In figure 2, the threshold field E'_c and the wave number Q_c are shown as the functions of the flexoelectric parameter v for different values of the parameter ε_φ. For the numerical calculations we used $\varepsilon_a=0.2$, $K=0.7\times10^{-6}$ dyn, the values of the flexoelectric coefficients e_1, e_3 were taken from the interval $(0.7/2.5)\times10^{-4}$ dyn$^{1/2}$.

As one can see, the electric field threshold value, E'_c, increases with increases in both the director azimuthal anchoring energy, ε_φ, and the flexoelectric parameter, v

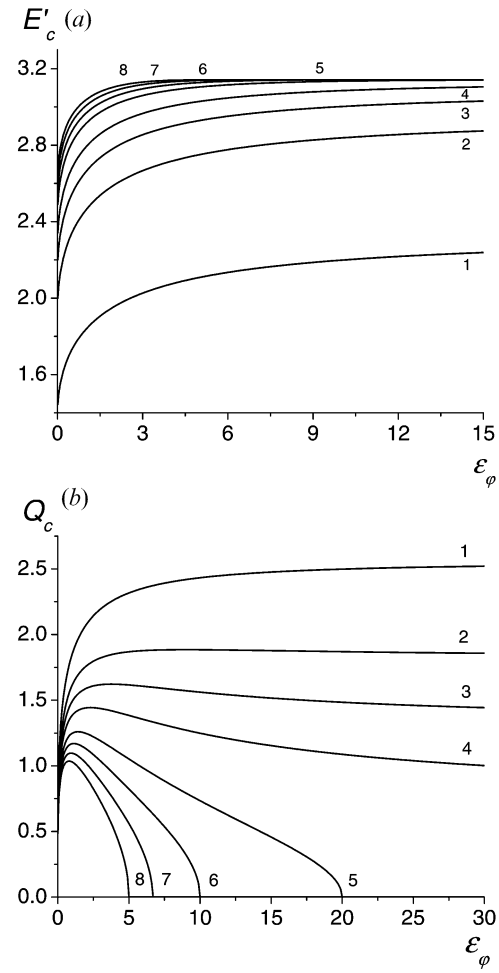

Figure 1. (a) Threshold field E'_c and (b) wave number Q_c versus the azimuthal anchoring energy ε_φ for different fixed values of v:v=0.2 (1); 0.5 (2); 0.7 (3); 0.9 (4); 1.2 (5); 1.4 (6); 1.6 (7); 1.8 (8).

(figures 1 a and 2 a). However, at $\varepsilon_\varphi \to 0$ the threshold electric field value E'_c approaches a constant value (at the fixed values of the parameter v), which is not equal to zero. It is conditioned by the fact that the director azimuthal deviation is determined not only by the value of W_φ, but the director reorientation angle in the plane xOz as well [see equation (7)]. Therefore the threshold field value can not be less than that determined by the polar anchoring energy value.

The period, $\lambda_c = \frac{2\pi}{q_c} = \frac{2\pi L}{Q_c}$, of the director spatial structure decreases monotonically with increasing ε_φ at the flexoelectric parameter values $0<v<\frac{1}{3}$ (figure 1 b). However, if the parameter $v>\frac{1}{3}$ the period λ_c of the director spatial structure depends on the anchoring energy ε_φ non-monotonically, i.e. with increasing ε_φ the quantity λ_c first decreases reaching some minimum value and further increases approaching to the value

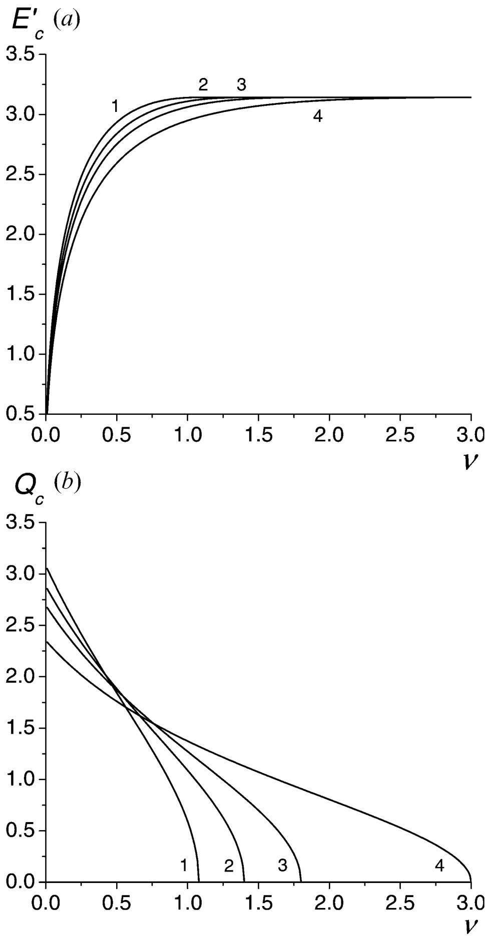

Figure 2. (a) Threshold field E_c' and (b) wave number Q_c versus the flexoelectric parameter ν for different fixed values of ε_φ:ε_φ=50 (1); 10 (2); 5 (3); 2 (4).

$\lambda_c^\infty = \frac{2\pi}{q_c^\infty}$, which is determined by equation (15) for $\nu<1$, or to the infinity if $\nu>1$.

From figure 1 b one can also see that for each value of the parameter $\nu>1$ the azimuthal anchoring energy critical value exists, namely, $\varepsilon_{\varphi th} = \frac{4}{\nu-1}$, in accordance with equation (22) (values of ε_φ, which correspond $Q_c=0$). If $\varepsilon_\varphi<\varepsilon_{\varphi th}$, the spatially periodic transition takes place, but if $\varepsilon_\varphi>\varepsilon_{\varphi th}$ only a uniform transition (along the Oy-axis) can be realized. At a fixed value of the parameter ε_φ, the period λ_c of the director spatial structure increases with increasing flexoelectric parameter ν (figure 2 b).

4. Influence of the polar anchoring energy

Now let us suppose the polar anchoring energy, W_θ, to be an arbitrary value, whereas the azimuthal anchoring energy equals infinity ($W_\varphi=\infty$). In this case the boundary conditions of equation (4) take the form

$$\left[(W_\theta \mp e_o E)\theta \pm K\frac{\partial\theta}{\partial z}\right]_{z=\pm L/2} = 0, \qquad (23)$$
$$\varphi|_{z=\pm L/2} = 0.$$

Substituting equation (9) into the boundary conditions of equation (23) we obtain, as in the previous case, the system of homogeneous algebraic equations for the coefficients a_i, b_i (i=1, 2). The condition of non-trivial solution to this system has the form

$$\left[AB - (e_o E)^2\left(\left(\in E^2\right)^2 + \left(2eEq\right)^2\right)\right] \cos\frac{p_1L}{2}\sin\frac{p_1L}{2}\cos\frac{p_2L}{2}\sin\frac{p_2L}{2} = 0, \qquad (24)$$

where

$$A = W_\theta\sqrt{(\in E^2)^2 + (2eEq)^2} - \frac{1}{2}Kp_1\left(\in E^2 + \sqrt{(\in E^2)^2 + (2eEq)^2}\right)\tan\frac{p_1L}{2} + \frac{1}{2}Kp_2\left(\in E^2 - \sqrt{(\in E^2)^2 + (2eEq)^2}\right)\tan\frac{p_2L}{2},$$
$$B = W_\theta\sqrt{(\in E^2)^2 + (2eEq)^2} + \frac{1}{2}Kp_1\left(\in E^2 + \sqrt{(\in E^2)^2 + (2eEq)^2}\right)\cot\frac{p_1L}{2} - \frac{1}{2}Kp_2\left(\in E^2 - \sqrt{(\in E^2)^2 + (2eEq)^2}\right)\cot\frac{p_2L}{2}. \qquad (25)$$

In the limiting case of the infinitely rigid director anchoring, $W_\theta=\infty$, we can obtain from equation (24) the results provided by equations (14)–(15). In the case of the strong polar anchoring ($\varepsilon_\theta = \frac{W_\theta L}{K} \gg 1$), the equations (24) and (8) can be solved, as in the case of the strong azimuthal anchoring, putting $p_1 = \frac{\pi}{L}(2s+1)(1-x)$, where $0<x<<1$. Limiting ourselves to the terms linear in $\frac{1}{\varepsilon_\theta}$ we obtain from equation (24)

$$p_1 = \frac{\pi}{L}(2s+1)\left(1 - \frac{1}{\varepsilon_\theta}\frac{\in E^2 + \sqrt{(\in E^2)^2 + (2eEq)^2}}{\sqrt{(\in E^2)^2 + (2eEq)^2}}\right), \qquad (26)$$

and, correspondingly, from equation (8) the dispersion dependence

$$E(q) = E^\infty(q)\left\{1 - \frac{2}{\varepsilon_\theta}\cdot\frac{\left(\frac{\pi}{L}(2s+1)\right)^2}{q^2 + \left(\frac{\pi}{L}(2s+1)\right)^2}\right\}, \qquad (27)$$

where $E^\infty(q)$ is defined in equation (13).

Minimizing equation (27) we obtain the threshold field value

$$E_c = E(q_c) = E_c^\infty\left(1 - \frac{1+v}{\varepsilon_\theta}\right), \qquad (28)$$

and the wave number of the director periodic structure, which equals

$$q_c = q_c^\infty\left(1 - \frac{1}{\varepsilon_\theta}\frac{1+v}{1-v}\right), \qquad (29)$$

if $v<1$ and $\varepsilon_\theta(1-v)>>1$, or

$$q_c = \frac{\pi}{\sqrt{2}L}\sqrt{1 - v - \frac{4}{\varepsilon_\theta}}, \qquad (30)$$

if $|1-v|<<1$.

Both the threshold field value, E_c, and the corresponding wave number, q_c, increase with increasing polar anchoring energy, ε_θ, at all permissible values of the flexoelectric parameter, v. From this one can see that the periodic structure of a director can only appear if the flexoelectric parameter v satisfies the inequality $v<1-\frac{4}{\varepsilon_\theta}$. However, for arbitrary values of ε_θ the general criterion of existence of the threshold periodic structure given by equation (22) does not come to the simple analytical expression, as in the case of the arbitrary azimuthal anchoring energy. In figure 3 we show the results of numerical calculation of the critical values of the parameter v in dependence on the values of ε_θ for some fixed values of the other flexoelectric parameter $v_o = \frac{\epsilon K}{e_o^2}$. Here $v<v_{th}$ determines the domain, where the spatially periodic structure of a director exists. As can be seen from figure 3, the finiteness of the polar anchoring energy results in a narrowing of the domain of the permissible values of the flexoelectric parameter v, in comparison with the case of the infinitely rigid director anchoring.

The threshold electric field E_c' and the corresponding wave number Q_c versus the polar anchoring energy ε_θ are shown in figure 4 for some values of the parameter v as a result of numerical calculation of equation (24). As in the case of the strong anchoring, the threshold field and the wave number increase monotonically with increasing anchoring energy ε_θ at all values of the flexoelectric parameter, v. At that, as one can see from figure 4 a, the threshold field $E_c' \to 0$ if $\varepsilon_\theta \to 0$ even at $\varepsilon_\varphi \to \infty$. It is easy to understand if one takes into account (see figure 4 b) that at small ε_θ a director reorients only in the plane xOz (unlike the case of small ε_φ and $\varepsilon_\theta=\infty$ considered above). The dependences of the threshold electric field E_c' and the wave number Q_c on the

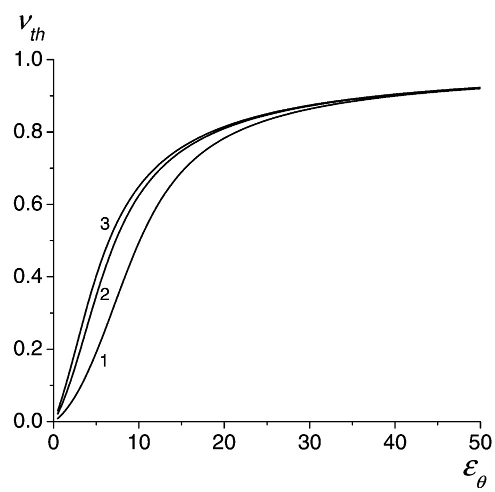

Figure 3. Critical value of the parameter v_{th} versus the polar anchoring energy ε_θ for different fixed values of v_o:v_o=0.1 (1); 0.3 (2); 0.5 (3).

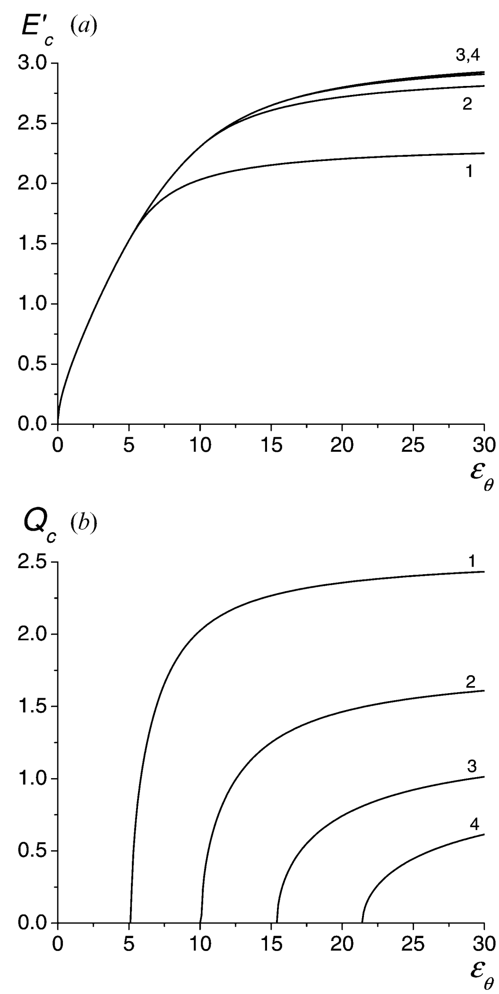

Figure 4. (a) Threshold field E_c' and (b) wave number Q_c versus the polar anchoring energy value ε_θ for v_o=0.1 and different fixed values of v:v=0.2 (1); 0.5 (2); 0.7 (3); 0.8 (4).

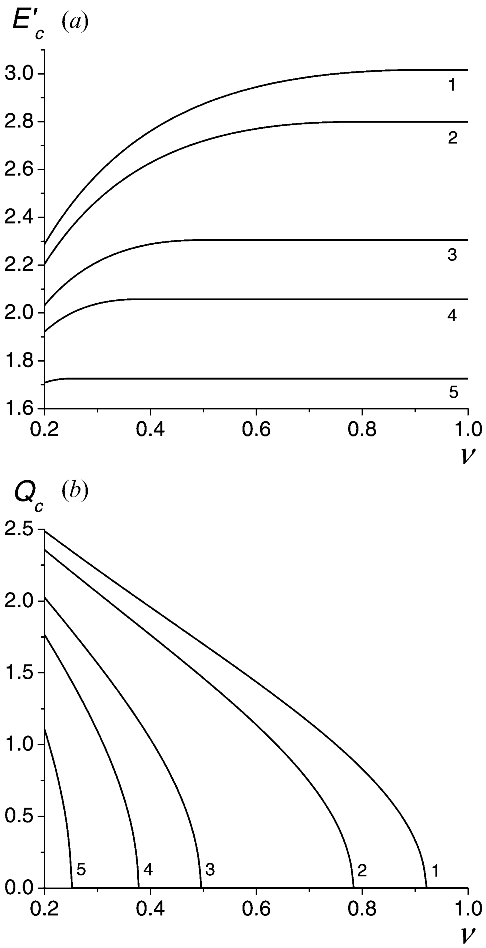

Figure 5. (a) Threshold field E'_c and (b) wave number Q_c versus the flexoelectric parameter v for v_o=0.1 and different fixed values of ε_θ:ε_θ=50 (1); 20 (2); 10 (3); 8 (4); 6 (5).

flexoelectric parameter v at the fixed values of the polar anchoring energy parameter ε_θ are shown in figure 5. At all numerical calculations we put the flexoelectric parameter v_o=0.1. Its values weakly influence the obtained dependencies $E'_c(\varepsilon_\theta)$, $Q_c(\varepsilon_\theta)$ and $E'_c(v)$, $Q_c(v)$.

5. Conclusions

The threshold electric field value, E_c, at which the spatially periodic reorientation of a director takes place, and the value of the director spatial period, λ_c, depend essentially on the director polar and azimuthal anchoring energy values. From this, E_c and λ_c depend more strongly on the polar anchoring energy value, W_θ, so as $W_\theta \to 0$ the threshold value $E_c \to 0$ and the director reorientation becomes uniform ($\lambda_c = \infty$), irrespective of the flexoelectric parameters values.

The domain of the values of the flexoelectric parameter, v, at which the spatially periodic reorientation of a director exists, depends on the anchoring energy value. In particular, in the case of the finite values of the azimuthal anchoring energy this domain enlarges, and in the case of the finite values of the polar anchoring energy it becomes more narrow in comparison with the case of the infinitely rigid director anchoring (which gives $v<1$). There is a significant difference with the case of homeotropic director anchoring, where the domain of the values of the flexoelectric parameter, v, at which the periodic reorientation of a director takes place, does not depend on the director anchoring energy value.

The influence of finite surface anchoring on the director spatially periodic reorientation in the planar nematic cell with non-equal elastic constants will be considered in a subsequent paper.

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